FTD-TT-63-232

TRANSLATION

IDEAS OF SOLUTIONS OF ALGEBRAIC EQUATIONS THROUGH DESIGNATED INTEGRALS

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FTD-TT- 63-232/1+2

UNEDITED ROUGH DRAFT TRANSLATION

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English Pages: 4

SOURCE: Ukrainian Periodical, Zbirnik Prats' z Obchyslyuval'noyi Matematyky i Tekhniky, Vol. 3, 1961, pp 42-44

S/696-61-3-0

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FTD-TT- 63-232/1+2 Date 22 April 19 63

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Ideas of Solutions of Algebraic Equations through Designated Integrals

by

B. N. Pshenichniy

The nature of the described by us method regarding the idea of solutions of algebraic solutions of equations by designation of integrals can be explained by utilizing the concept of generalized δ-function.

In accordance with designations

$$\int_a^b \delta(x) f(x)\,dx = \begin{cases} f(0) & 0 \in (a, b), \\ 0 & 0 \bar{\in} (a, b). \end{cases} \qquad (1)$$

Assuming that we have an algebraic equation

$$f(x) = 0 \qquad (2)$$

and it is known, that in the interval (a,b) is situated only one root of this equation. Assuming that in this intervals exists a derivative $f'(x)$, different by it from zero. The function $y = f(x)$ has a reverse function $x = x(y)$.

We will calculate the integral

$$\int_a^b x\delta[f(x)]\,|f'(x)|\,dx = \operatorname{sign} f(b) \int_{f(a)}^{f(b)} x(y)\,\delta(y)\,dy = x(0) \qquad (3)$$

In accordance with designations of inverted function, x(0) - root of equation (2). In this way, we will obtain a formula

$$x_0 = \int_a^b x\delta[f(x)]\,|f'(x)|\,dx, \qquad (4)$$

which formally offers the root of equation (2). But this formula is unsuitable for practical calculation, since for the calculation of its right side, in accordance with designations of δ- function, it is first necessary to know the root.

To make equation (4) effective, we will utilize other designations of δ-function as weak boundary of a certain regular sequence of an ordinary function. There can be many such sequences. As an example we could take from them such two

$$f_n(x) = \frac{\sqrt{n}}{\sqrt{\pi}} e^{-nx^2} \quad (n \to \infty); \qquad (5)$$

$$f_\alpha(x) = \frac{\alpha^{2m-1}}{\beta} \frac{1}{\alpha^{2m} + x^{2m}} \,(\alpha \to 0),\ \beta = \int_{-\infty}^{+\infty} \frac{dx}{1 + x^{2m}}. \qquad (5')$$

Utilizing (5) or (5'), we will obtain formulas

$$x_0 = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{\pi}} \int_a^b x e^{-nf^2} |f'(x)|\, dx; \tag{6}$$

$$x_0 = \lim_{\alpha\to 0} \frac{\alpha^{2m-1}}{\beta} \int_a^b x \frac{|f'(x)|}{\alpha^{2m} + f^{2m}(x)}\, dx. \tag{6'}$$

Next we will utilize the sequence (5'), figuring, that it is possible to utilize any given sequence, which fits weakly to δ-function.

Formulas (6) without any characteristics are generalized for the system of equations:

$$f_i(x_1, \dots x_n) = 0 \quad i = 1, \dots, n. \tag{7}$$

Since in zone D is situated only one solution (7) and the Jacobian systems in that zone differ from zero, then

$$x_i^0 = \lim_{\alpha\to 0} \frac{\alpha^{n(2m-1)}}{\beta^n} \int_D n \int x_i \prod_{j=1}^{n} (\alpha^{2m} + f_j^{2m})^{-1} |J|\, dx_1 \dots dx_n. \tag{8}$$

We will write formula (8) in case of a linear system. To do this we will first calculate the integral

$$\frac{\alpha^{n(2m-1)}}{\beta^{-n}} \int_{-\infty}^{+\infty} n \int \prod_{j=1}^{n} \left[\alpha^{2m} + \left(\sum_{i=1}^{n} a_{ji} x_i - b_j\right)^{2m}\right]^{-1} dx_1 \dots dx_n =$$

$$= \frac{1}{|D|} \frac{\alpha^{n(m-1)}}{\beta^n} \int_{-\infty}^{+\infty} n \int \prod_{j=1}^{n} (\alpha^{2m} + z_j^{2m})^{-1} dz_1 \dots dz_n = \frac{1}{|D|}. \tag{9}$$

where D - determinant of matrix $\{a_{ji}\}$ n. Utilizing equation (9), we will obtain

See page 2a for Equation 10

If in the integrals in (10) are made substitutions of variables

$$x_i = x_i^0 + y_i. \quad 10a$$

then one can become convinced, that their ratios do not depend upon α. Finally for the linear system we will obtain a formula

$$x_k^0 = \frac{\int_{-\infty}^{+\infty} n \int x_k \prod_{j=1}^{n} \left[1 + \left(\sum_{i=1}^{n} a_{ji} x_i - b_j\right)^{2m}\right]^{-1} dx_1 \dots dx_n}{\int_{-\infty}^{+\infty} n \int \prod_{j=1}^{n} \left[1 + \left(\sum_{i=1}^{n} a_{ji} x_i - b_j\right)^{2m}\right]^{-1} dx_1 \dots dx_n}. \tag{11}$$

$$x_k^0 = \lim_{\alpha \to 0} \frac{\int\limits_{-\infty}^{+\infty} n \int x_k \prod\limits_{j=1}^{n} \left[\alpha^{2m} + \left(\sum\limits_{i=1}^{n} a_{ji} x_i - b_j \right)^{2m} \right]^{-1} dx_1 \ldots dx_n}{\int\limits_{-\infty}^{+\infty} n \int \prod\limits_{j=1}^{n} \left[\alpha^{2m} + \left(\sum\limits_{i=1}^{n} a_{ji} x_i - b_j \right)^{2m} \right]^{-1} dx_1 \ldots dx_n},$$

$$m > 1. \qquad (10)$$

Equation 10

The derived formulas can be very effectively utilized for practical calculation, when the integrals, which are included in them, are expressed through an elementary function, as for example, in such a simple case.

Given are equations

$$\cos x = 0$$

It is known, that on the intervals $(0, \pi)$ lies only one of its roots. Utilizing formula (6') we will obtain

$$x_0 = \lim_{a \to 0} \frac{a}{\pi} \int_0^{\pi} \frac{x \sin x dx}{a^2 + \cos^2 x}, \quad 11a$$

If we calculate the integral in this formula, making a substitution $x = \pi - t$

$$\int_0^{\pi} \frac{x \sin x dx}{a^2 + \cos^2 x} = -\int_0^{\pi} \frac{t \sin t dt}{a^2 + \cos^2 t} + \pi \int_0^{\pi} \frac{\sin t dt}{a^2 + \cos^2 t}; \quad 11b$$

In this way we will obtain $\int_0^{\pi} \frac{x \sin x dx}{a^2 + \cos^2 x} = \frac{\pi}{2} \int_0^{\pi} \frac{\sin t dt}{a^2 + \cos^2 t} = \frac{\pi}{2a}\left[\operatorname{arctg} \frac{1}{a} - \operatorname{arctg} \frac{-1}{a}\right].$

$$x_0 = \lim_{a \to 0} \frac{a}{\pi} \cdot \frac{\pi}{2a}\left[\operatorname{arc tg} \frac{1}{a} - \operatorname{arc tg} \frac{-1}{a}\right] = \frac{\pi}{2}. \quad 11c$$

Since integrals are not taken, then, by selecting sufficiently low α, is possible to calculate the integral by the Monte-Carlo method. This will give us approximated values of the sought for root.

In conclusion the authors wish to express great thanks to V.E.Shamanskiy for valuable advice and aid in this report.

Given are formulas expressing a solution of any given algebraic equations in form of definite integrals. If these integrals are taken, the root can be found accurately. If they are not taken it is possible to find an approximated value of the root, calculating the obtained integrals by some approximation method, e.g. by the Monte-Carlo method.

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